Name and Surname : Solve

Grade/Class : 11/..... **Mathematics Teacher** : .....

150

ANSWER BOOKLET

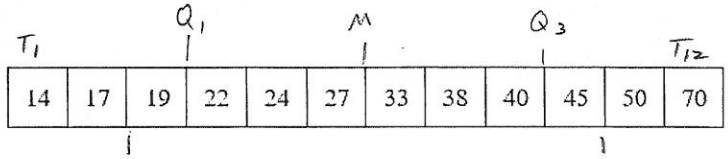
Grade 11

November Paper 2

November 2020

QUESTION 1

1.



1.1.1.	$M = \frac{27+33}{2}$	
	$= 30$ ✓	1
1.1.2.	$Q_1 = \frac{19+22}{2}$ $\quad Q_3 = \frac{40+45}{2}$	
	$= 20,5$ ✓both $\quad = 42,5$	
	$\therefore IQR = 42,5 - 20,5$	
	$= 22$ ✓	2
1.1.3.	$\bar{x} = 33,25$ ✓	1
1.1.4	$\sigma = 15,62$ ✓	1

12.	$\bar{x} \pm \sigma = 33,25 \pm 15,62$	
	$= 17,63$ or $48,87$ ✓ both	
	$\therefore \frac{8}{12} \times 100 = 66,67\ \%$ ✓	2
13.	$\bar{x} - M = 33,25 - 30$	
	✓ $= 3,25 > 0$ ✓	
	$\therefore$ data is positively skewed (OR) skewed to the right	2
14.	upper fence $= Q_3 + 1,5.IQR$	
	$= 42,5 + 1,5.22$	
	$= 75,5$ ✓	
	$70 < 75,5$	
	$\therefore$ 70 will NOT be an outlier ✓	2

QUESTION 2

2.1.

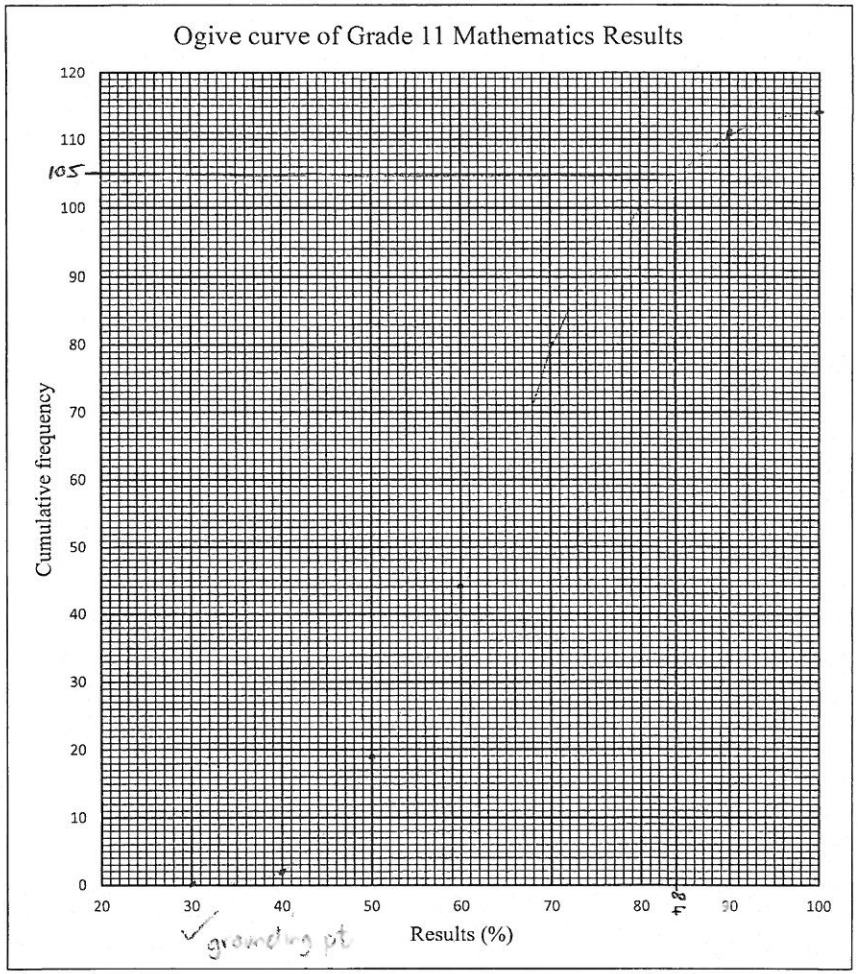
Mark %	Frequency	Cumulative frequency
$30 < x \leq 40$	2	2
$40 < x \leq 50$	17	19
$50 < x \leq 60$	25	44
$60 < x \leq 70$	36	80
$70 < x \leq 80$	20	100
$80 < x \leq 90$	11	111
$90 < x \leq 100$	3	114

✓ 1

2.2.	$60 < x \leq 70$ ✓	1
2.3.	$\bar{x} = \frac{35 \times 2 + 45 \times 17 + \cdots + 95 \times 3}{114}$ midpt ✓ × f ✓	
	$= \frac{7290}{114}$	
	$= 63,77\ \%$ ✓	3
2.4.1.	$T_1; \ldots; T_{114}$ $M = T_{\frac{1}{2}(1+114)}$	
	$= T_{57,5}$	
	$T_{58}; \ldots; T_{114}$ $Q_3 = T_{\frac{1}{2}(58+114)}$	
	$= T_{86}$ ✓	1

2.4.2.	$P_{60} = T_{\frac{60}{100}(14+1)}$	
	$= T_{69}$ ✓	1

2.5.



✓ upper end of int. when plo

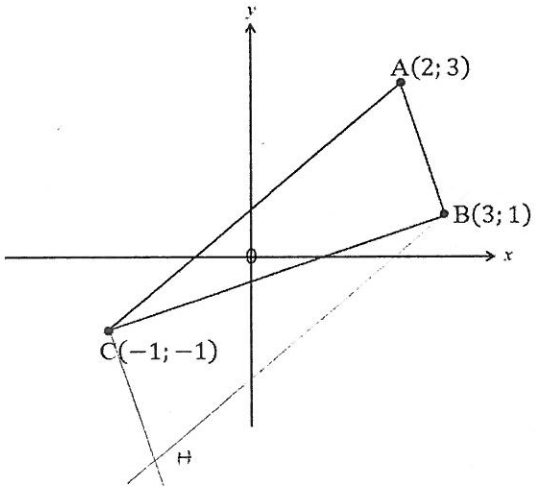
✓ smooth curve

3

2.6.	$\leq 84\% = 105$ ✓	
	$\therefore > 84\% = 114 - 105$	
	$= 9$ learners ✓	2

QUESTION 3

3.



3.1.1.	$AB = \sqrt{(1-3)^2 + (3-2)^2}$ ✓ A(2;3) B(3;1)	
	$= \sqrt{5}$ ✓	2
3.1.2.	$AD = \sqrt{(d-3)^2 + (3,5-2)^2}$ ✓ A(2;3) D(3,5;d)	
	$= \sqrt{(d-3)^2 + \frac{9}{4}}$	
	$AB = AD$	
	$\sqrt{5} = \sqrt{(d-3)^2 + \frac{9}{4}}$	
	$5 = (d-3)^2 + \frac{9}{4}$ ✓ $()^2$ bs	
	$\frac{11}{4} = (d-3)^2$ ✓	
	$\pm\sqrt{\frac{11}{4}} = d-3$ ✓	

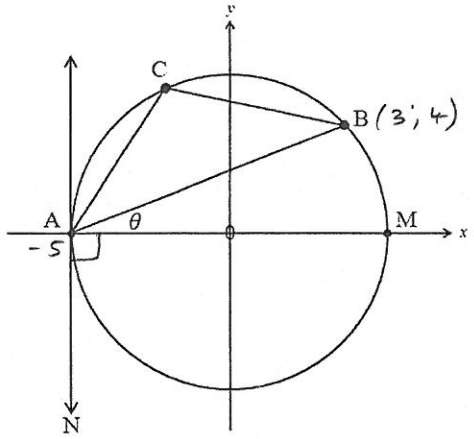
(AD)
✓ $2d^2 - 24d + 25 = 0$
✓ QF
✓ both s's

	$d - 3 = 1,65\ldots$ or $d - 3 = -1,65\ldots$	
	$d = 4,66$ ✓both $d = 1,34$	5
3.2	$m_{AB} = \frac{1-3}{3-2}$ A(2;3) B(3;1)	
	$= -2$ ✓	
	$m_{BC} = \frac{-1-1}{-1-3}$ B(3;1) C(−1;−1)	
	$= \frac{1}{2}$ ✓	
	$m_{AB} \times m_{BC} = (-2) \times (\frac{1}{2})$ ✓	
	$= -1$ ✓	4
	$\therefore A\hat{B}C = 90°$	
3.3.	$m_{AE} = \frac{-8-3}{e-2}$ A(2;3) E(e;−8)	
	$= \frac{-11}{e-2}$ ✓	
	$\therefore -2 = \frac{-11}{e-2}$ ✓equate $m_{AB} = -2$ (3.2.)	
	$-2(e-2) = -11$	
	$-2e + 4 = -11$	
	$e = \frac{15}{2}$ ✓	3
3.4	$m_{AC} = \frac{-1-3}{-1-2}$ A(2;3) C(−1;−1)	
	$= \frac{4}{3}$ ✓	

	$y = \frac{4}{3}x + c$	
	sub $A(2;3)$	
	$3 = \frac{4}{3} \cdot 2 + c$ ✓	
	$\frac{1}{3} = c$	
	$\therefore y = \frac{4}{3}x + \frac{1}{3}$ ✓	
	$\times 3: \quad 3y = 4x + 1$	
	$-4x + 3y - 1 = 0$ ✓	4
3.5.	$y = \frac{4}{3}x + c$ ✓ $\quad m_{BF} = m_{AC}$ ‖	
	sub $B(3;1)$	
	$1 = \frac{4}{3} \cdot 3 + c$ ✓	
	$-3 = c$	
	$\therefore y = \frac{4}{3}x - 3$ ✓	3
3.6.	conv $\angle$ in semi $\odot = 90°$ ✓	1
3.7.	$A(2;3) \xrightarrow[4 \downarrow]{3 \leftarrow} C(-1;-1)$	
	$B(3;1) \xrightarrow[4 \downarrow]{3 \leftarrow} H(0;-3)$ ✓ ✓	2

QUESTION 4

4.

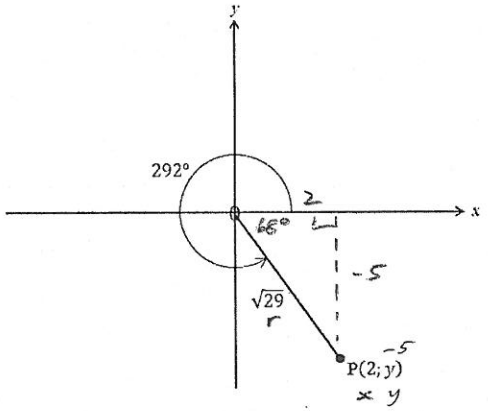


4.1.1.	tan ⊥ rad ✓	1
4.1.2.	$x = -5$ ✓	1
4.2.1.	$m_{AB} = \frac{4-0}{3-(-5)}$ $\quad A(-5;0)\ B(3;4)$	
	$= \frac{1}{2}$	
	$\therefore \tan\theta = \frac{1}{2}$ ✓	
	ref $\angle = 26,56\ldots°$	
	tan + in	

	I : $\theta = 26,57^\circ$ ✓	2
4 2.2.	$N\hat{A}B = 90^\circ + 26,57^\circ$	
	$= 116,57^\circ$ ✓ R	
	$\therefore A\hat{C}B = 116,57^\circ$ ✓✓ ^ tan chord	2
4.3.	$x_M = \frac{(-5)+(3)}{2}$ $y_M = \frac{(0)+(4)}{2}$ A(-5;0) B(3;4)	
	$= -1$ $= 2$	
	$\therefore (-1;2)$ ✓✓	2

QUESTION 5

5.1.



5 1.1.	$(2)^2 + y^2 = (\sqrt{29})^2$ Pythag	
	$y^2 = 25$	
	$y = \pm\sqrt{25}$ reject +	
	$y = -5$ ✓	1
5 1.2.	(a) $\sin 292^\circ = \frac{y}{r}$	
	$= \frac{-5}{\sqrt{29}}$ ✓	1
	(b) $\cos(-292^\circ) = +\cos 292^\circ$	
	$= \frac{x}{r}$	
	$= \frac{2}{\sqrt{29}}$ ✓	1

5.1.2. (c) $360° - 292° = 68°$ "S in area" = 360°

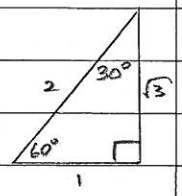
$\tan 68° = \frac{o}{a}$

$= \frac{5}{2}$ ✓ NB +5 → 1

5.2. $\frac{\sin 197° \tan 300° \cos(297° + x)}{\cos(-x - 150°) \cos 107°}$

• $\sin 197°$
$= \sin(180° + 17°)$
$= -\sin 17°$

• $\tan 300°$
$= \tan(360° - 60°)$
$= -\tan 60°$
$= -\frac{\sqrt{3}}{1}$



• $\cos(297° + x)$
$= \cos(90° + x)$
$= -\sin x$

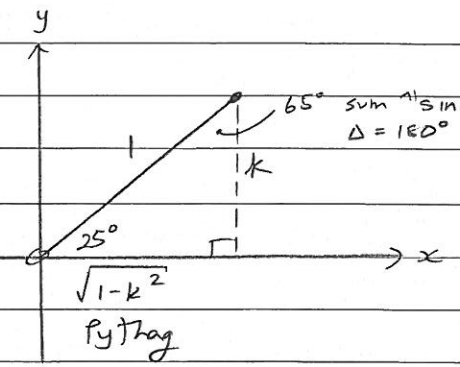
• $\sin(-x - 180°)$
$= \sin(-x + 180°)$
$= \sin(180° - x)$
$= \sin x$

• $\cos 107°$
$= \cos(90° + 17°)$
$= -\sin 17°$

$\therefore \frac{(-\sin 17°)(-\sqrt{3})(-\sin x)}{(\sin x)(-\sin 17°)} = \sqrt{3}$ → 6

5.3.1. $\sin 25° = k$

$= \frac{k}{1}$ $\frac{y}{r}$



y, r ✓
x ✓

$\tan 25° = \frac{y}{x}$

$= \frac{k}{\sqrt{1 - k^2}}$ → ✓ 3

5.3.2. $\tan 65° = \frac{o}{a}$

$= \frac{\sqrt{1-k^2}}{k}$ → ✓ 1

QUESTION 6

6.1.1. $\left(\frac{1}{\cos x} - \tan x\right)^2 = \frac{1-\sin x}{1+\sin x}$

$LHS = \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$ ✓

$= \left(\frac{1-\sin x}{\cos x}\right)^2$ ✓

$= \frac{(1-\sin x)^2}{\cos^2 x}$

$= \frac{(1-\sin x)^2}{1-\sin^2 x}$ ✓

$= \frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$ ✓

$= \frac{1-\sin x}{1+\sin x}$

$= RHS$ ✓ 4

6.1.2. ID not valid when $(k \in \mathbb{Z})$

- $\cos x = 0$: $x = 90° + k180°$ ✓
- $\tan x = UD$: $\frac{\sin x}{\cos x} = UD$, $\cos x = 0$, as before
- $1 + \sin x = 0$: $\sin x = -1$, $x = 270° + k360°$ ✓ 2

6.2.1. $2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$ref\angle = 30°$

$\sin -$ in $(k \in \mathbb{Z})$

III : ✓ $x = 210° + k360°$

IV : ✓ $x = 330° + k.360°$ 2

6.2.2. $\sin(x - 20°) + \cos 2(x + 30°) = 0$

$A = x - 20°$ $B = 2(x + 30°)$

$\sin A + \cos B = 0$

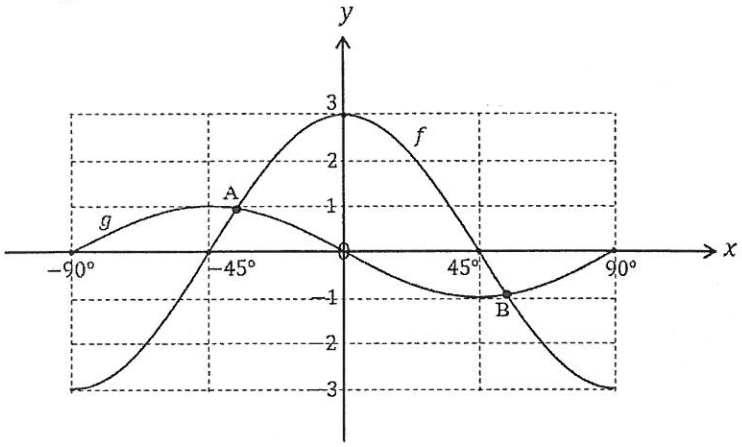
$\sin A = -\cos B$ ✓ $(k \in \mathbb{Z})$

III $\sin(270° - B)$ $\sin(270° + B)$ IV

$\sin A = \sin(270° - B)$ or $\sin A = \sin(270° + B)$

$A = 270° - B + k360°$ ✓	$A = 270° + B + k360°$ ✓
$x - 20° = 270° - 2(x + 30°) + k360°$	$x - 20° = 270° + 2(x + 30°) + k360°$
$x - 20° = 270° - 2x - 60° + k360°$	$x - 20° = 270° + 2x + 60° + k360°$
$3x = 230° + k360°$	$-x = 350° + k360°$
$x = 76,67° + k.120°$ ✓	$x = -350° - k.360°$ ✓ 5

6.3. $f: y = 3\cos 2x$ $g: y = -\sin 2x$



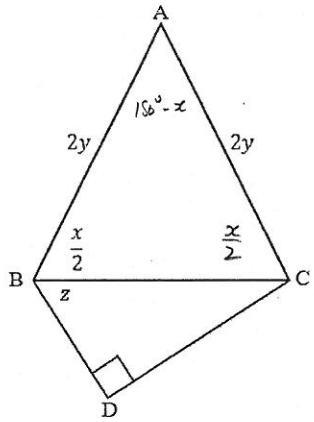
6.3.1.	(a) Amplitude g = 1 ✓	1
	(b) Period $f = \frac{360°}{2}$	
	$= 180°$ ✓	1
	(c) Range g: $y \in [-1; 1]$	
	$x - 28°$ $y \in [-1; 1]$	
	$\times 2$ $y \in [-2; 2]$	
	-3 $y \in [-5; -1]$ ✓ ✓	2

6.3.2.	where $f \cap g$: $3\cos 2x = -\sin 2x$ ✓	
	$A = 2x$ $3\cos A = -\sin A$	
	$\div \cos A$ $\frac{3\cos A}{\cos A} = \frac{-\sin A}{\cos A}$	
	$3 = -\tan A$	
	$-3 = \tan A$ ✓	
	$\text{ref}^\angle = 71,56\ldots°$	
	tan − in $(x \in \mathbb{Z})$	
	II: $A = 108,43\ldots° + k180°$	
	$2x = 108,43\ldots° + k180°$	
	$x = 54,21\ldots° + k90°$ ✓	
	$\therefore x_B = 54,22°$ ✓ and $x_A = -35,78°$ ✓	5
6.3.3.	(a) $g(x) - f(x) > 0$	
	$g(x) > f(x)$	
	$y_g > y_f$	
	$\therefore x \in [-90°; -35,78)$ or $(54,22°; 90°]$ ✓ ✓	2
	· if x_A, x_B are estimated from graph ie not using (6.3.2.) award full marks	

6.3.3.	(b) $f(x).g(x) \leq 0$	
	$y_f \cdot y_g \leq 0$	
	$\therefore x \in [-90°; -45°]$ or $[0°; 45°]$ or $x = 90°$ ✓ ✓ ✓	3
6.3.4	g : $y = -\sin 2x$	
	i : $y = \sin(2x + 80°)$	
	$= \sin 2(x + 40°)$	
	g was	
✓	• translated 40° horizontally to the left	
✓	• reflected in the x-axis	
	to become i.	2

QUESTION 7

7.

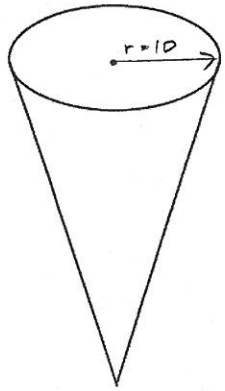


7.1.	$A\hat{C}B = \frac{x}{2}$ ✓SR ∠'s opp = sides	
	$\hat{A} = 180° - (\frac{x}{2} + \frac{x}{2})$ sum ∠'s in $\Delta = 180°$	
	$= 180° - x$ ✓SR	2
7.2.1.	area ΔABC	
	$= \frac{1}{2}(2y)(2y)\sin(180° - x)$ ✓	
	$= 2y^2(\sin x)$	
	$= 2y^2 \sin x$	1

7.2.2.	$BC^2 = (2y)^2 + (2y^2) - 2(2y)(2y)\cos(180° - x)$ ✓	
	$= 4y^2 + 4y^2 - 8y^2(-\cos x)$	
	$= 8y^2 + 8y^2\cos x$	
	$= 4y^2(2 + 2\cos x)$ ✓	
	$BC = \pm\sqrt{4y^2(2+2\cos x)}$ reject −	
	$= \sqrt{4y^2}\sqrt{2+2\cos x}$	
	$= 2y\sqrt{2+2\cos x}$ ✓	
	$\sin z = \frac{CD}{BC}$ ✓	
	$CD = BC.\sin z$	
	$= 2y\sqrt{2+2\cos x}.\sin z$	
	$= 2y\sin z\sqrt{2+2\cos x}$	5

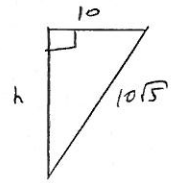
QUESTION 8

8.1.



CM

$TSA = 100(\sqrt{5}+1)\pi$



	$TSA = \pi r^2 + \pi r h_s$	
	$100(\sqrt{5}+1)\pi = \pi(10)^2 + \pi(10)h_s$ ✓	
	$100\pi\sqrt{5} + 100\pi = 100\pi + 10\pi h_s$	
	$10\sqrt{5} = h_s$ ✓	
	$h^2 + 10^2 = (10\sqrt{5})^2$ ✓c/s Pythag	
	$h^2 = 400$	
	$h = \pm\sqrt{400}$ reject −	
	$= 20$ ✓	

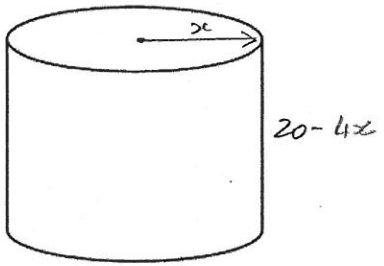
$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (10)^2 . 20$ ✓

$= 2094,40 \ cm^3$ ✓ 6

- don't penalise missing units

8.2.



cm

8.2.1

$TSA = 2\pi r^2 + 2\pi r.h$

$= 2\pi(x)^2 + 2\pi(x)(20-4x)$ ✓ both terms

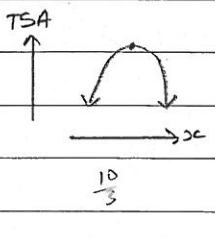
$= 2\pi x^2 + 2\pi x(20-4x)$

$= 2\pi x^2 + 40\pi x - 8\pi x^2$ ✓ x out

$= -6\pi x^2 + 40\pi x$ 2

8.2.2

$TSA = -6\pi x^2 + 40\pi x$

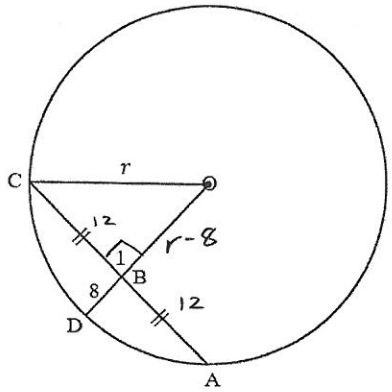


$x_{tp} = \frac{-(40\pi)}{2(-6\pi)}$ ✓

$= 3,33 \ cm$ ✓ 2

QUESTION 9

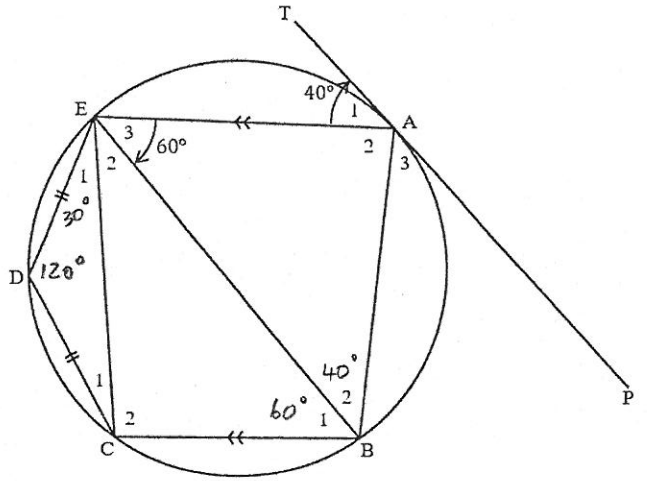
9.



9.1.	line from centre O to midpt of chord ✓	1
9.2.	$OB = r - 8$ radii	
	$r^2 = (r-8)^2 + 12^2$ ✓SR fy thag	
	$r^2 = r^2 - 16r + 64 + 144$	
	$16r = 208$	
	$r = 13$ ✓	2

QUESTION 10

10.



10.1.	$\hat{B}_3 = 40°$ ✓S ✓R ∠ tan chord	2
10.2.	$\hat{B}_1 = 60°$ ✓S ✓R alt ∠'s =, AE ∥ BC	2
10.3.	$\hat{D} = 120°$ ✓S ✓R opp ∠'s cyclic quad = 180°	2
10.4.	Let $\hat{E}_1 = x$	

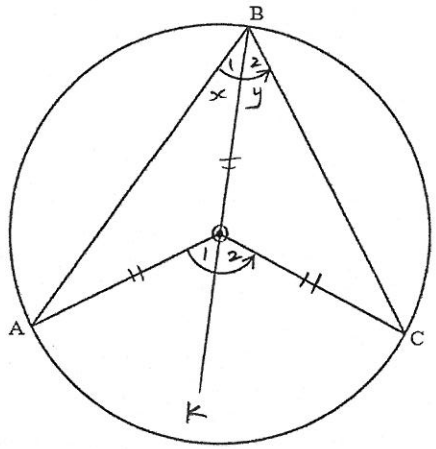
$\therefore \hat{C}_1 = x$ ✓SR ^'s opp = sides

$2x + 120° = 180°$ ✓SR sum ^'s in $\Delta = 180°$

$x = 30°$ 2

QUESTION 11

11.1.



✓constr

Constr: BO extended to K

Let $\hat{B}_1 = x$ $\hat{B}_2 = y$

$\hat{O}_1 = \hat{A} + x$ ✓SR Ext ^ Δ

but $\hat{A} = x$ ✓SR radii, ^'s opp = sides

$\therefore \hat{O}_1 = 2x$ ✓

Similarly, $\hat{O}_2 = 2y$

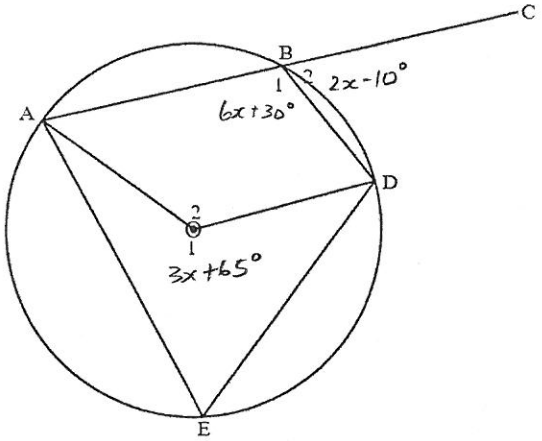
$A\hat{O}C = \hat{O}_1 + \hat{O}_2$ ✓

$= 2x + 2y$

$= 2(x+y)$

$= 2 . A\hat{B}C$ 5

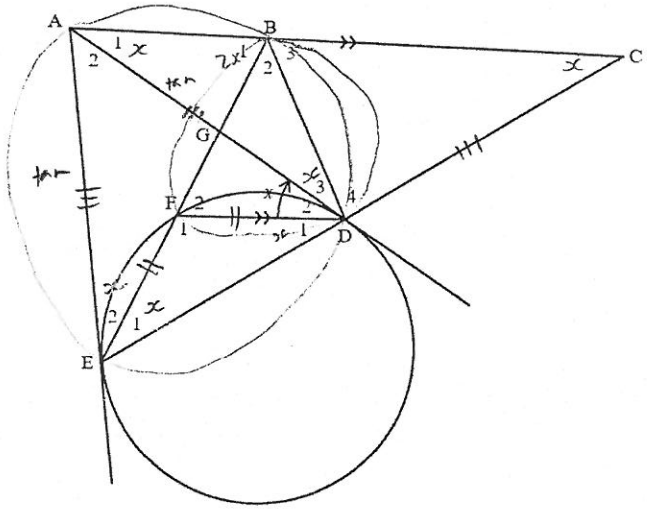
11.2.



	$\hat{B}_1 = \frac{1}{2}(3x+65°)$ ✓ ^ @ centre = 2 ^ @ circum.	
	$= \frac{3}{2}x + 32,5°$ ✓	
	$\frac{3}{2}x + 32,5° + 2x - 10° = 180°$ ✓SR ^'s on str line = 180°	
	$\frac{7}{2}x = \frac{315}{2}$	
	$x = 45°$ ✓	4

QUESTION 12

12.



12.1.	$\hat{E}_1 = x$	✓S ✓R ^ tan chord	
	$\hat{A}_1 = x$	✓SR alt ^'s =, AC ∥ FD	
	$\therefore \hat{A}_1 = \hat{E}_1$	both = x	
	$\therefore$ ABDE is a cyclic quad	✓R conv ^'s in same segm =	4
12.2.1.	$\hat{D}_1 = x$	✓ ^'s opp = sides	
	$\therefore \hat{C} = x$	✓ corr ^'s =, AC ∥ FD	
	$\therefore \hat{C} = \hat{A}_1$	both = x	

	$\therefore$ CD = AD ✓SR sides opp = ^'s	
	AD = AE ✓S ✓R tan's from ext common	
	pt =	
	$\therefore$ AE = CD ▶ both = AE	5
12.2.2.	$\hat{E}_2 = x$ ✓SR ^ tan chord	
	$\therefore \hat{D}_3 = x$ ✓S ✓R ^'s in same O segm =	
	$\hat{B}_1 = 2x$ ✓SR ^'s in same O segm =	
	But $\hat{D}_2 + \hat{D}_3 = 2x$	
	$\therefore \hat{D}_2 + \hat{D}_3 = \hat{B}_1$ both = 2x	
	$\therefore$ ABC is a tan ✓R converse ^ tan chord	
	to ⊙ BFD ▶	5